Control of the Triana Spacecraft in the Presence of Fuel Slosh Dynamics

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Triana is a mission to the LaGrange neutral gravity point (L1) between the Earth and the Sun, due to be launched in the year 2001. Three months after leaving the Earth, Triana will enter its final orbit about the L1 point and begin collecting scientific data. To support this mission, the on-board Attitude Control System (ACS) utilizes five distinct feedback controllers. Two of the five controllers, Delta-H and Delta-V, use thrusters to provide attitude correction torques, where Delta-H handles vehicle momentum management and Delta-V delivers commanded trajectory corrections. These two thruster control loops differ only by commanded target and exit criteria; thus, they admit identical linear analyses.

Since the Triana propulsion system is fueled by liquid hydrazine, the net thruster forces exerted on the spacecraft will excite fuel slosh. These slosh dynamics were neglected in the initial controller design, and there is concern that the controller may react adversely with the slosh. This paper develops the dynamics necessary to model the fuel slosh. Linear analysis examines the interaction between the slosh dynamics and the controller, identifying potential instabilities and quantifying controller robustness.

Introduction

The Triana observatory is a mission dedicated to helping scientists construct more accurate models of the Earth's climate and to studying the effects of solar radiation on this climate. Triana will be placed in an orbit about the LaGrange neutral gravity point (L1) between the Earth and the Sun. This vantage will allow the observatory named after Rodrigo de Triana, the lookout who first saw the New World from Columbus' ship, a continual view of the lit face of the Earth.

To accomplish this mission, the on-board Attitude Control System (ACS) uses five different control modes. Three of the controllers, Sun-Acquisition, Science, and Safehold, are reaction wheel driven, differing mainly in sensor complement. Two of the controllers, Delta-H and Delta-V, utilize the thrusters to maintain vehicle attitude. The on-board propulsion system consists of hydrazine thrusters. Sufficiently large external forces exerted on the spacecraft will excite slosh within the liquid hydrazine tank, possibly adversely affecting the controller. While the vehicle is guided by one of the three reaction wheel based controllers, the external forces are limited to minute environmental disturbances, such as solar pressure and gravity gradient. Fortunately, these environmental forces are too small to excite fuel slosh. However, during the thruster based control modes, the net propulsive force on the body may be of sufficient authority to excite the slosh dynamics.

The two thruster based controllers, Delta-H and Delta-V, are essentially identical controllers. The Delta-H mode uses eight of the thrusters to maintain vehicle attitude and null rates while the reaction wheels spin down. This unloads the vehicles stored momentum and frees reaction wheel control authority. The Delta-V attitude controller functions identically, except that two additional delta-v thrusters are used deliver the commanded change in vehicle velocity. These two delta-v thrusters function completely separately from the attitude control thrusters. Thus, the Delta-H and Delta-V attitude controllers admit identical linear analyses. When the initial linear design was completed, there was insufficient information to properly model the slosh phenomenon, and thus the analysis neglected the fuel dynamics. This paper develops the appropriate dynamical models and considers the robustness and stability of the thruster based control loop including the effects of slosh dynamics.

Description of Triana

The Triana observatory in the deployed configuration is shown in Figure 1. The main body consists of three subsections: the Propulsion Module (PM), which houses the 28" diameter hydrazine tank and propulsion system components; the SMEX (SMall EXplorers) Lite Spacecraft Bus, containing electronic boxes and other subsystem components; and the Observatory Upper Structure, which supports additional boxes and components, as well as the instrument payload.

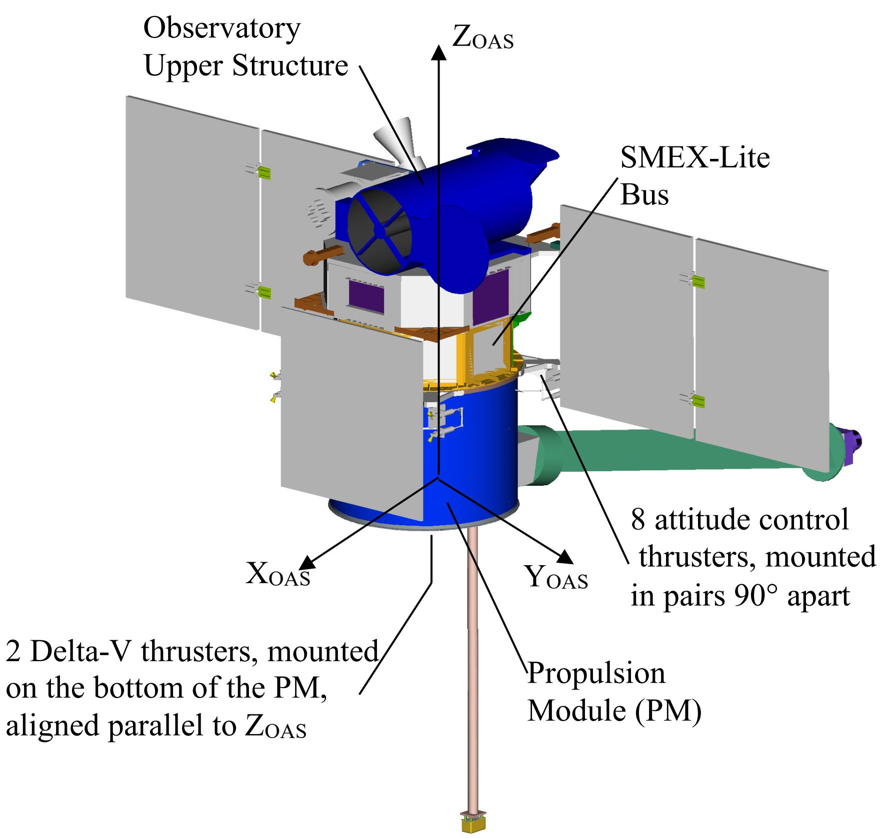


Figure 1: Triana Observatory, Deployed Configuration

The spherical hydrazine tank contains a diaphragm, a flexible membrane, which assures that the propellant remains in contact with the propellant port and aids damping of sloshing motion. The tank is mounted with its centerline coincident with the Z-axis, placing the diaphragm perpendicular to the Z-axis. Eight of the hydrazine thrusters are mounted at the top of the PM. These are fired in pairs to provide attitude control torques, but with a near zero resultant force on the body. On the bottom of the PM, two additional hydrazine thrusters are mounted parallel to the Z-axis. These are fired simultaneously, to provide large changes in velocity with a near zero resultant torque. During Delta-V, these two thrusters deliver trajectory adjustments, while the remaining eight maintain attitude control. For momentum management, the Delta-H controller uses only the eight attitude control thrusters.

Modeling Fuel Slosh.

Before developing a model for the Triana tank, a few general comments pertaining to slosh modeling are in order. Propellant slosh is a mechanical effect of liquid propellants. Where the fuel motion impacts the wall of the tank, it will transfer momentum to the vehicle. To understand this phenomenon, it is necessary to derive an appropriate fuel slosh model for the particular tank configuration and vehicle layout under study.

Most slosh models mimic the fuel slosh frequency using pendulums or masses with springs and dampers whose frequency and placement on the spacecraft are dependent on a number of factors, including spacecraft acceleration, tank geometry, and fuel fill level. These models are based on the observation that the forces and moments exerted on a tank as predicted by ideal, irrotational, potential flow are similar in form to those predicted by a system of pendulums. Bryson provides an excellent treatment of the simplest pendulum slosh model, approximating the slosh in a spherical tank by considering the fuel as a lumped mass, sliding along the interior surface of the tank. This simple pendulum model must be heavily modified if there are diaphragms, baffles or separate compartments within the tank or if the tank is non-spherical.

Once the lumped mass model has been properly applied to the situation under, the parameters such as spring and damping constants must be derived. The most common approach involves experimental study, where a scale model of the tank is excited and the response measured. Then, the parameters for the lumped mass model may be fit to the empirical data. If experimental data is not available for the actual tank under study, these factors may be scaled from existing empirical data provided the data applies to a tank of near identical geometry and excitation sources.

Not only are the slosh models extremely dependent on the particular vehicle and tank geometry, but, it must be noted that sometimes these lumped fuel models are completely inadequate. For example, if the tank is not axially symmetric about the nominal thruster vector, a simple pendulum model is often insufficient and additional dynamics must be modeled. The Stardust spacecraft provides an excellent example of an alternative approach. The spacecraft's hydrazine tank was positioned off-axis due to science requirements, a geometry that greatly complicated the slosh dynamics. A constrained surface fuel model was developed and subsequently verified using computational fluid dynamics program, Flow-3D (Flow Science, Inc.).

Modeling the Triana Tank.

The intent of this paper is to study the effect of fuel slosh during the two thruster control modes, Delta-H and Delta-V. During thruster control, the attitude control is accomplished by firing pairs of thrusters to create torque couples. However, misalignment and variations in thruster forces will inevitably create some small net forces acting on the body, f_x and f_z, exciting the fuel slosh. These are applied at the thruster attach point, a distance, l_x and l_z from the vehicle center of mass. This situation is illustrated in Figure 2. The vehicle is treated as a rigid body, with a mass and inertia consistent with an empty tank. The fuel mass is treated separately.

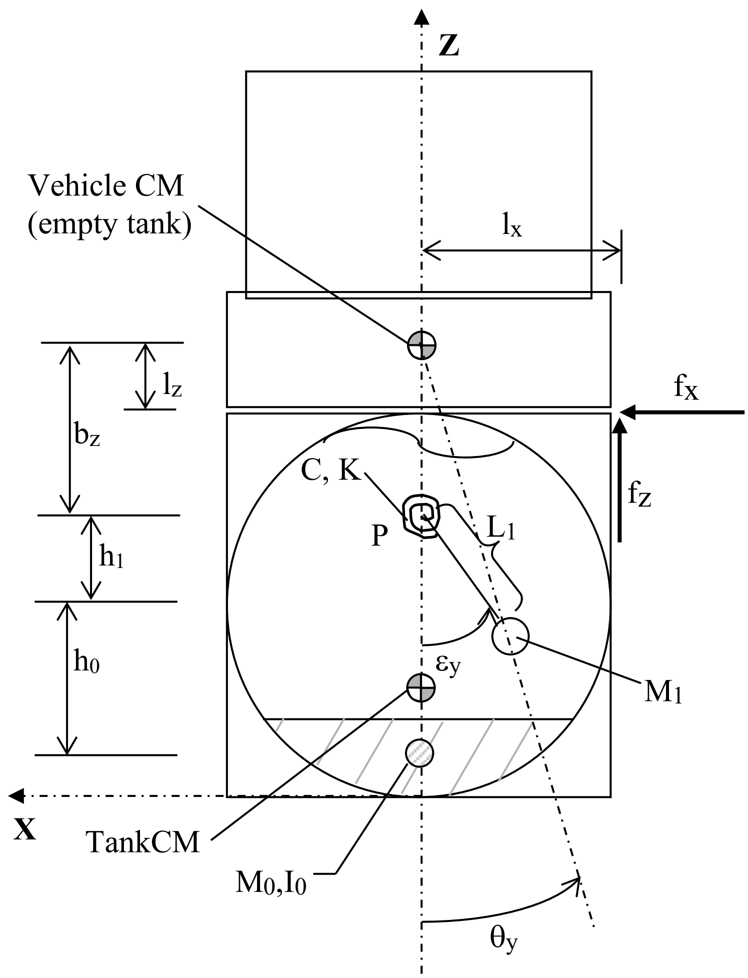
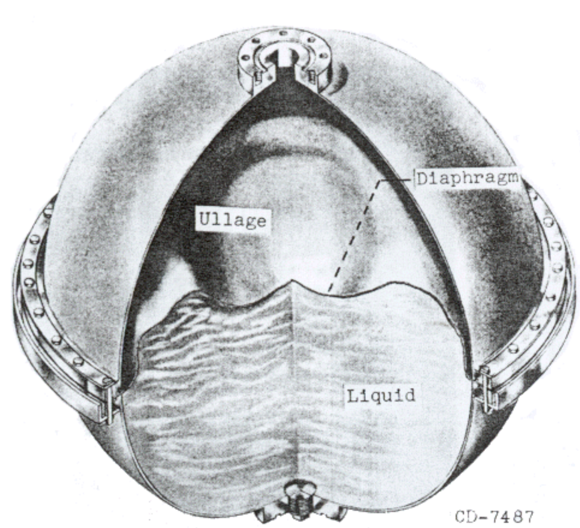


Figure 2: Triana Spacecraft & Simplified Fuel Model

The Triana tank is a Pressure Systems, Inc., 28-inch spherical pressure vessel constructed of 6Al-4V titanium. Positive fuel expulsion is provided by a reversible ethylene-propylene terpolymer (AF-E-332) rubber diaphragm retained (welded-in) at the sphere mid-plane. Mounting is accomplished on polar bosses. Figure 3 shows a tank of identical configuration.

An extensive experimental study of the TDRS tank, a tank of similar configuration to the Triana tank, suggests a more complex model than the simple pendulum used by Bryson. To account for the diaphragm effects, the fuel mass (M_f) is divided into two parts. A pendulous mass (M_1) is used to represent the liquid participating in the sloshing motion, and an immobile mass (M_0) models the remainder of the liquid. The pendulum pivot height, h_1, is chosen so that the sloshing



torques created by a rotational oscillation about the Y-axis are duplicated, and the immobile mass is assigned a centroidal moment of inertia, I_o, mimicking the rigid body rotation of the liquid. This immobile mass is positioned along the vertical centerline (h_0) such that the model's center of mass location matches the center of mass for the liquid. The primary difference between this model and a simple pendulum slosh model is the use of a torsional spring (K) and dashpot (C) at the pendulum pivot. This simulates the bladder restraint on the liquid

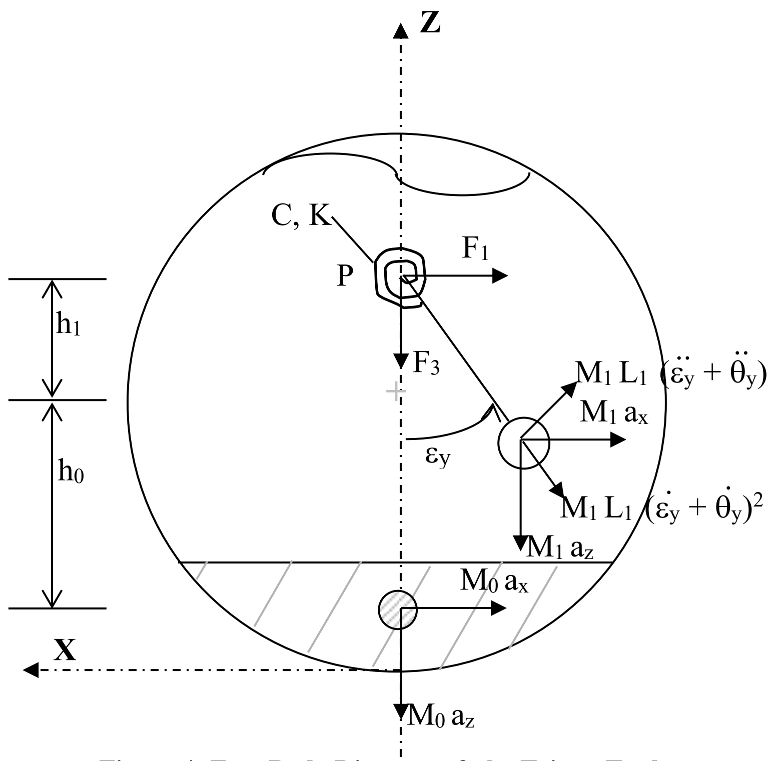
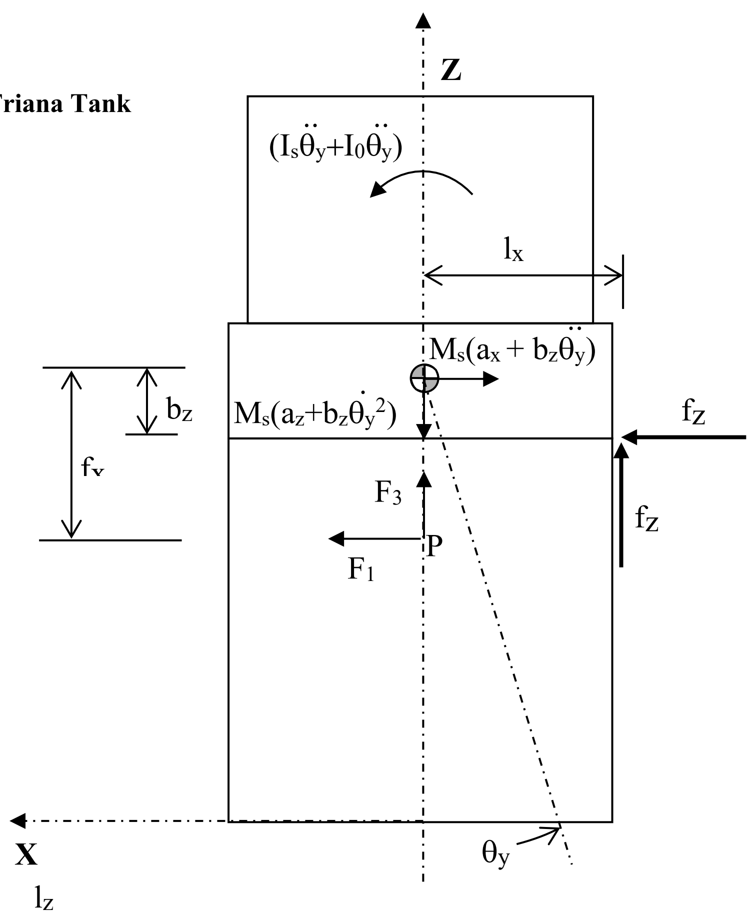


Figure 4: Free Body Diagram of the Triana Tank

motion, where K simulates the stiffening effect of the bladder on the slosh frequency and C represents the total damping of the sloshing. The free body diagram for the fuel mass is shown in Figure 4. The matching free body diagram for the spacecraft is sketched as Figure 5. The point, P, is the pendulum attach point. F_1 and F_3 represent the support forces for the pendulum.

Because of symmetry, a similar model is valid for the Y-Z plane. Given that the goal is to study the effect of the slosh dynamics on the controller, that the controller is identical in all axes, and that the cross products of inertia are quite small, single axis linear analysis is sufficient. Further, a rate limit is placed on all slew maneuvers, ensuring rates will always be small.



With small cross products and small body rates, the coupling term, , of the rigid body dynamics, , is almost zero, and the vehicle dynamics are essentially uncoupled. Therefore, we restrict our study to planar motion, and the equations of motion are written for the spacecraft and fuel dynamics accordingly.

The equations of motion for the fuel mass are easily written from the free-body diagram:

Similarly, the equations of motion for the spacecraft are:

Finally, the kinematics equations can be written as:

Derivation of the Model's Constants

Several of the parameters present in the equations of motion depends solely on Triana's physical properties, and remain fixed regardless of fuel fill level, FL. The spacecraft mass and inertia with an empty tank, $I_s = 264$ kg-m^2 and $M_s = 492$ kg, remain fixed. Also, the distance to the thruster, l_x=0.52 m stays constant. The spring and damping constants, K and C, are functions of the diaphragm thickness. The remaining parameters, l_z, M_0, I_0, M_1, L_1, h_1, h_0, and b_z vary with the fuel fill level. The vertical distance from the center of mass to the thruster attach point, l_z, varies linearly from 0.066 m at maximum fill level (72%) to 0.19 m for an empty tank: meters.

The spring and damping constants are extrapolated in S. Sirlin's work from the data provided by A. Stofan. Stofan's investigation of the slosh-damping properties of diaphragms in spherical tanks under excitations similar to those expected for Triana provides empirical data for C and K that can be scaled based on diaphragm thickness. The Triana tank has a diaphragm approximately one-

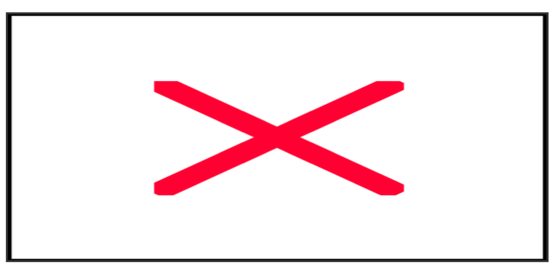


Table 1: Diaphragm Constants

sixteenth of an inch (0.0625") thick, for which the experimental data predicts a K=54.75 Nm/rad and C=4.569 Nms/rad.

Expression are derived for the remaining parameters, which vary as a function of fill level. Using data from the NEAR and TDRS studies, dimensionless curves are estimated for M_1/M_f, L_1/R and h_1/R. Both the TDRS and NEAR tanks were of the same configuration as shown in Figure 3, but of different size. The TDRS and NEAR slosh data were non-dimensionalizing, by dividing the masses by the total fuel mass, M_f, and the lengths by the tank radius, R. The M_1/M_f curve (Figure 6) shows the expected general trend: At higher fill levels less fuel participates in the sloshing motion. Note that the data is imprecise. Theoretically, at 100% fill, all fuel moves as a rigid body, i.e., M_1=0, an expected trend which disagrees with the M_1/M_f curve. The L_1/R and h_1/R curves, shown in Figure 7, also exhibit trends matching the expected physics. The length of the sloshing pendulum, L_1, is the 'shortest', i.e., the sloshing frequency is the highest, when the tank is approaching empty, a trend which agrees with Dodge's work on the TDRS tanks. Negative values for h_0 and h_1, indicate that the location of the masses, M_0 and M_1, are on the opposite sides of the tank than assumed in the Figures 4 and 5 drawings. Empirical data shows that the attach point (h_1) is generally positive when the tank is above half full, and negative when then tank is more than half empty. This is caused by the mass distribution. As the tank empties, the tank's center of mass shifts downward causing the location of M_0 and M_1 to likewise migrate along the -Z axis. However, the h_1/R curve fit diverges from this trend as the tank empties, falsely indicating a suddenly positive h_1 at fill levels below 15%. This is an artifice of the data-fitting algorithm, and the h_1/R curve should not be used at lower fill levels. Still, the agreement between the curve fit and the empirical data is excellent at the higher fill levels providing sufficient data to characterize controller/slosh interaction.

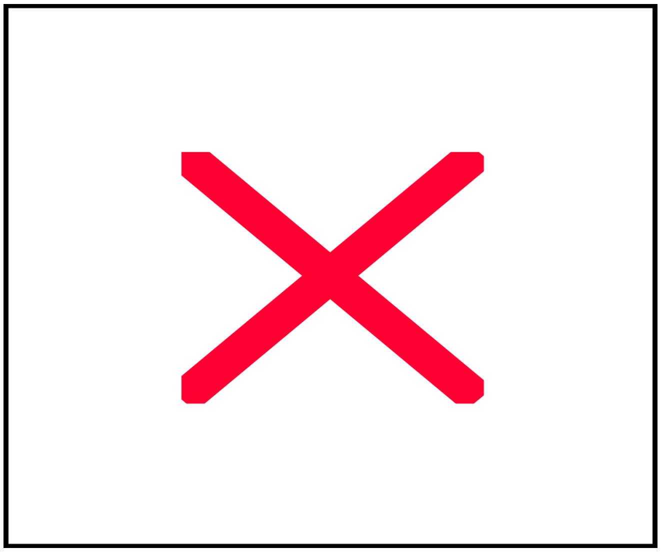
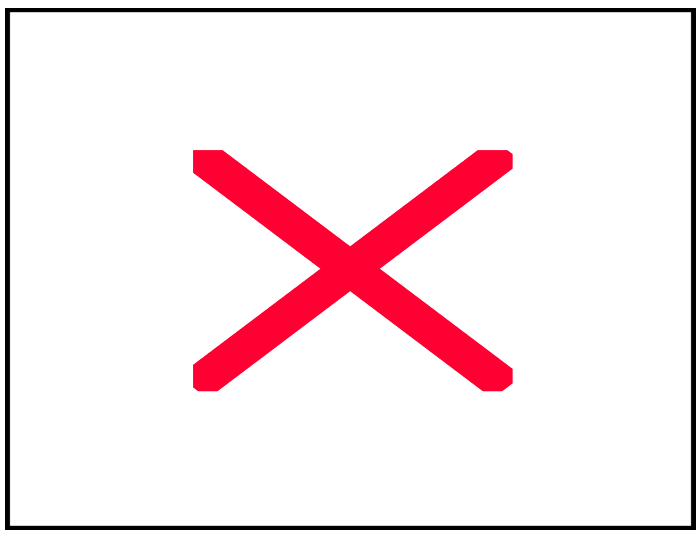


Figure 6: M_1/M_f Curve



The curves presented in Figures 6 and 7 can be approximately applied to the Triana tank by multiplying by Triana's total fuel mass, which is a function of fill level, and Triana tank radius, which is a constant R = 0.3556 m, respectively.

To find the total mass as a function of percent full, FL, an approximate expression for M_f is found from the geometry of the tank by assuming the fuel fills some portion of a sphere, and noting that the maximum fuel height is theoretically 28"=0.7112 m:

.

The density of hydrazine is approximated as , which neglects all variations due to temperature. It is interesting to notice that this value matches the density of water, explaining why so many hydrazine slosh tests use water as the test fluid. The actual fuel shape is constrained by the diaphragm, but this approximation produces results good to within 5%.

M_1 is found by applying the polynomial fit given in Figure 6 and multiplying the corresponding M_f. Finally, M_0 is found by differencing the total fuel mass, M_f, and M_1.

Once the sloshing and fixed masses and the pendulum and attach point lengths are found, the location of the fixed mass, h_0, is dictated by matching the actual center of mass to the model's center of mass. Simple calculations give the tank's center of mass in meters, measured from the bottom of the tank:

Whereas, the model's center of mass is:

Equating these expressions yields a solution for the only unknown, h_0:

The final parameter, I_0, is found from the liquid torque acting on the tank as a function of excitation frequency. The data available is sparse. The TDRS experiment only considered two rotational excitation cases, and scaled a linear fit to the two points. Non-dimensionalize this data by the empty tank inertia, and a coarse approximation for I_0 can be derived. From the experimental data, I_0', the inertia of the 'rigid' fuel mass about an axis through the M_0 lump, is estimated as: I_0' can be scaled to the Triana based on I_T=8.16 kg-m^2. To find the inertia of the 'rigid' fuel mass about the center of the tank, the parallel axis theorem is applied: .

Interaction of the Slosh Dynamics with the Attitude Control System

Now that the model parameters are available, the next task it to derive the appropriate transfer functions to characterize the effect of fuel slosh. Consider the rigid body equations of motions derived previously. Eliminating F_1, F_3, a_x, and a_z leads to the following expressions:

where the combined terms are defined as:

For , and small deviations from equilibrium, the Laplace transform of the linearized equations of motion is:

The transfer functions from f_x to θ_y and from f_z to θ_y may be derived from the above relations:

The values of the transfer function coefficients vary with of fill level. Based on manufacturer's guidelines, the Triana tank is rated for a maximum fill level of 72%. At this fill level, the transfer functions become

Both slosh modes share the same poles, where the two poles at the origin represent the rigid body dynamics. The complex poles indicate a slosh mode with a natural frequency of 8.77 rad/sec (1.4 Hz) with a damping ratio of 0.46. Also, notice the two complex zeros. Their natural frequencies are 9.55 rad/sec and 16.6 rad/sec. This places one set of the complex zeros quite close to the slosh mode poles, which will mitigate the effect of the slosh mode. These values, of course, change over mission life as the fill level decreases. This variation of parameters will be studied in the robustness analysis.

Analyzing the Thruster Controller

The thruster controllers' attitude loop for both Delta-V and Delta-H utilizes a PD controller to calculate a commanded torque which is applied to the spacecraft by firing pairs of the attitude thrusters to produce pure torque couples. The flight software calculates new control torques for all three axes at a rate of 10 Hz. Here, analysis is restricted to the single-axis attitude control about the Y-axis. In practice, the commanded control torques are converted to thruster pulses using a pulse width modulator. For this analysis, the pulse width modulator and matching jet select logic are modeled as the ideal conversion between torque and force. Since the Triana thrusters are placed symmetrically about the control axes, the f_z and f_x are nominally equal. To achieve a forceless torque, the thrusters are fired in matched pairs, creating torque couples. This leads to the conversion factor used in the linear analysis: $Trq = 2(f_z l_x + f_x l_z)= 2f(l_x + l_z)$.

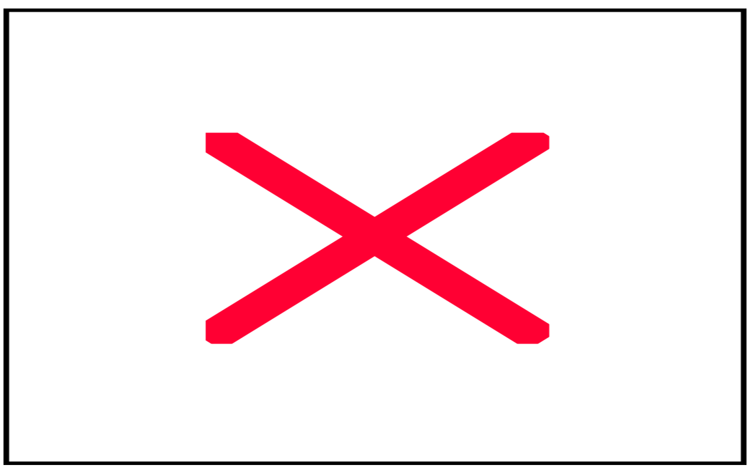


Figure 8: Thruster Mode Linear Analysis Block Diagram

Note it is assumed that any commanded torque can be achieved, ignoring finite quantization from the pulse width modulation for this first order analysis. The analog logic of the Engine Valve Driver (EVD) card adds a delay of 10 Hz: The card is driven by the receipt of a fire command. A command is received, telemetry on the current state of the EVD components is gathered, then the fire command is processed, entailing a delay which is conservatively taken as an entire control cycle. Resulting rate and attitude are sensed by a gyro that outputs whole angles at a 200 Hz rate with no electronic filtering. Compared to the 10 Hz sampling rate, the gyro transfer function may be approximated as an unity gain and gyro dynamics may be omitted from the linear analysis. Due to coupling inherent in the thruster configuration, only one body axis can be controlled at a time, forcing the thruster command to cycle between the X, Y and Z control axes. This effect neglected in this simplified continuous analysis: A more precise discrete analysis would properly capture the potential phase loss of this phenomenon.

There are two plants that may be used in the linear analysis. In the idealized dynamical model, the spacecraft responds only as a rigid body and thruster firings do not excite the fuel slosh. This leads to a nominal plant containing only rigid body dynamics.

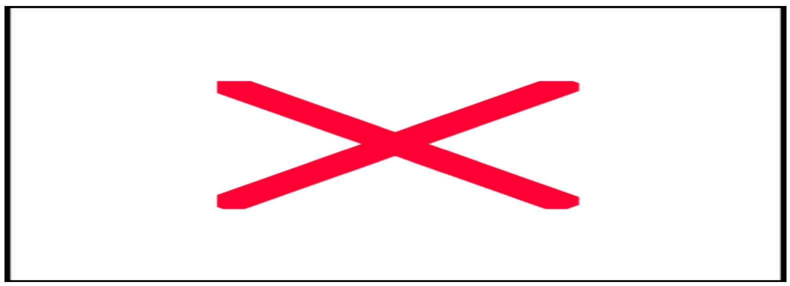


Figure 9: Nominal and True Plant

However, the true plant includes the slosh dynamics, using the transfer functions previously derived. A misalignment gain is also included in the true plant. Ideally, the thruster components, f_x and f_z, are equal and the misalignment gain is unity. However, slight misalignments in the hardware mounting will skew the thrust vectors. In addition, small variations in the thruster force will occur with each firing. These effects cause the thrust vector to be slightly off nominal in both magnitude and direction, so that f_x and f_z are no longer equal. With an expected mounting accuracy of ±0.25° and a thruster force deviation of ±5%, the misalignment gain may vary from 0.95 to 1.05.

Considering these two plants, the nominal and true attitude control loop can be rewritten

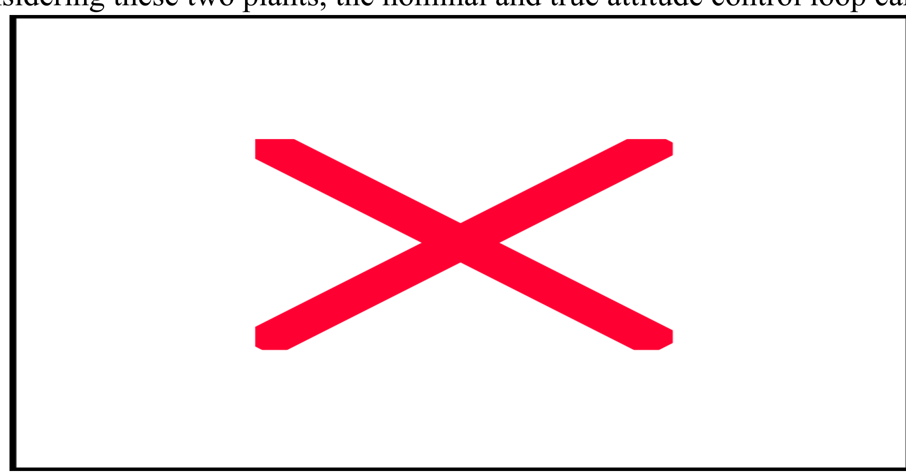


Figure 10: Nominal and True Plant

with the nominal loop including only the rigid body response and the true loop includes the slosh dynamics. In this analysis, the sample and hold operation shown in Figure 8 is modeled by the continuous transfer function .

Linear Analysis

The first step is to compare the nominal loop, containing only rigid body dynamics, to the true loop, including rigid and slosh dynamics. The open loop frequency response for the nominal and true loop are shown in Figure 11.

Figure 11: Nominal and True Open Loop Frequency Response and Stability Margins

The general practice followed by the Triana design team require stability margins of at least 6 dB gain and 30° phase. The controller gains of K_p=38 Nm/rad and K_r=108 Nms/rad show excellent relative stability in the nominal loop, with a bandwidth of ~0.5 rad/sec. This results in a frequency separation between the controller dynamics and the slosh frequency of ~9 rad/sec of over a decade. Yet, there may be cause for concern. That healthy nominal gain margin of 27 dB is measured at a frequency of 11 rad/sec, close to the slosh mode frequency. However, when we examine the true loop's frequency response, surprisingly, the effect of the slosh mode is barely visible. The complex zeros at 10.4 rad/sec mostly cancel the slosh dynamics, capturing the physical effect of the

Nominal Open Loop		**True Open Loop***					
Zeros	Poles	Zeros	ω_n (r/s)	ζ	Poles	ω_n (r/s)	ζ
20	0	20	-	-	0	-	-
20	0	20	-	-	0	-	-
-0.354	-20	-0.354	-	-	-3.8±j8.4	9.2	0.46
		-4.8±j9.2	10.4	0.46	-20	-	-

*Uses Misalignment of Unity: The misalignment gains are always so close to unity (0.95 to 1.05), that the effect of the misalignment term to is insignificant.

Table 2: Open Loop Dynamics at 72% (Max) Fill Level

diaphragm. These zeros primarily come from the C and K terms in the transfer function numerator, terms which capture the diaphragm damping. The linear analysis confirms that the diaphragm has successfully suppressed the fuel slosh. It also confirms that estimating the plant as a rigid body is adequate for controller design, at least until structural dynamics are quantified. Note, both loops include right-half plane zeros, a factor of the delay.

This is a satisfying justification for propulsion management devices. To further illustrate this point, contrast the dynamics of Triana's true plant with the unrestrained fuel slosh examined by Bryson. Bryson found a true plant of the form: . These plant dynamics have a pair of zeros on the imaginary axis, a pair of poles at the origin and a pair of poles on the imaginary axis. Root locus theory shows us that if the zeros lie above the poles in Bryson's plant, the system is inherently unstable. To illustrate this compare the two root loci shown in Figures 12 and 13.

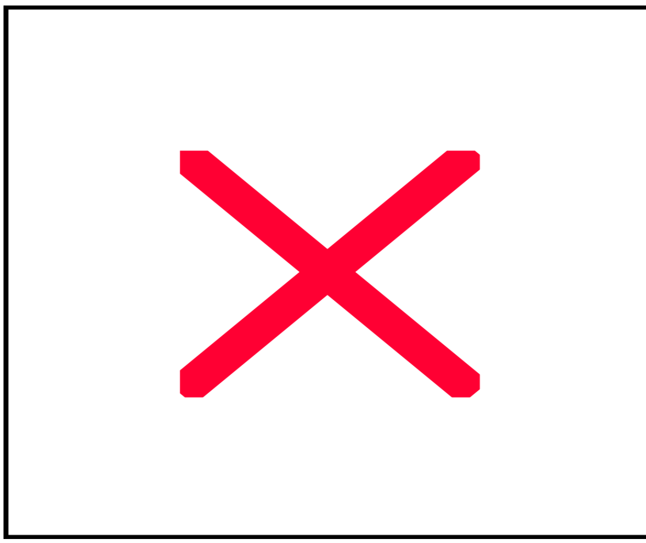


Figure 12: Bryson's Plant Root Locus

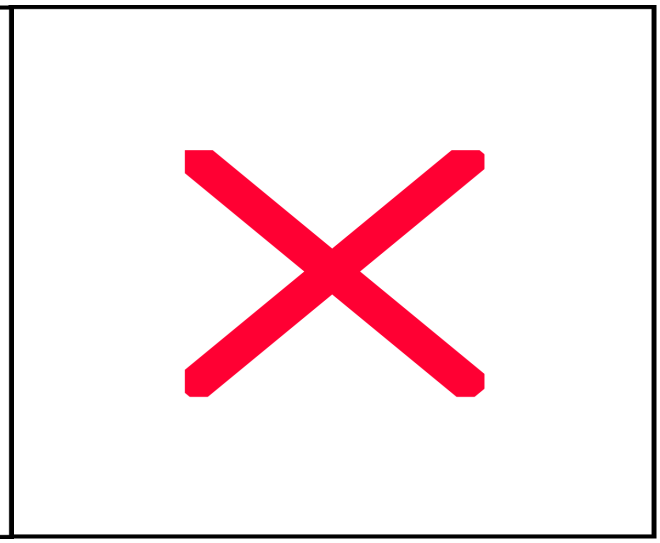


Figure 13: Triana Plant Root Locus

Figure 13 is the root locus of the true Triana open loop using the transfer function given in Figure 10. This includes the slosh suppression effects of the tank's bladder. Compare this to Figure 12 which shows the root locus of the Triana control loop, except Bryson's model of unsuppressed fuel slosh is substituted for the Triana plant dynamics.

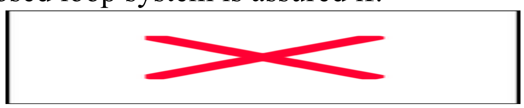
In Bryson's case, nothing damps the fuel slosh. The root locus shows that a gain of 0.005 will drive the system unstable, indicating that even with a gain reduction, the system is unstable. Now consider Figure 13, showing the root locus for the Triana plant. Notice the poles and zeros of the slosh mode are well away from the imaginary axis. A gain increase of 30 is required to push the slosh poles toward the right-half plane; and, this instability is only caused by the delay, as the slosh poles migrate toward the open-loop right-half plane zero produced by the delay. Without the delay, there would be no chance of destabilizing the slosh mode.

The frequency response analysis revealed a pole-zero cancellation that showed that the restraint of the bladder suppresses the slosh dynamics. This root locus comparison compares restrained and unrestrained slosh behavior, showing that the bladder provides the crucial dynamics to produce system stability.

Robustness.

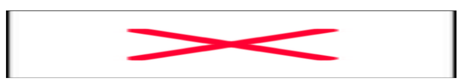
The linear analysis presented above concentrated on the beginning of life case at the 72% fill level. However, the question of robustness remains. Over the mission life, as the fill level decreases, vehicle mass properties will shift and the true plant will change. The controller design which provides adequate stability at the beginning of life may have different properties by the end of life. To examine robustness, the multiplicative error model, also known as the "fuzzy ball" uncertainty model is applied. This test quantifies the difference between the 'true' and 'nominal' open loop transfer functions as:

Assuming the uncertainty transfer function, Δ(s), has no right-hand plane poles, then the stability of the true closed loop system is assured if:



To apply this test, the uncertainty transfer function is derived from our knowledge of the true and nominal open loop transfer functions:

Robustness may be ascertained by plotting this robustness boundary against the



frequency response of the nominal closed loop transfer function. To quantify stability over mission lifetime, the robustness boundary is plotted for various fill levels.

Figure 14: Robustness Boundary as Function of Fill Level

As the fill level decreases, the robustness boundary shifts steadily downward. The boundary for the 72% fill level displays a large transient at the slosh mode frequency. This transients flattens out as the fuel level falls, and appears to be approaching a asymptote along the +4 dB line.
In all cases, the fuzzy-ball test guarantees stability of the true closed loop system, indicating the attitude controller will remain stable over the life of the mission.

This is a gratifying results. However, the robustness boundary test is only calculated from the beginning of life fill level of 72% to the 20% fill level. As noted in the models parameter derivation, experimental data is insufficient to derive accurate fuel slosh model values at fill levels below 20%. In the literature, it is generally assumed that slosh is neglible at these low fuel levels, especially with the bladder restraint. However, the change in fuel will still affect the vehicle inertia, changing the rigid body properties. The vehicle's empty inertia of 227 kg-m^2 is obtained by analytical calculation. Linear analysis of the nominal loop indicates gain and phase margins of 27 dB and 50°, nearly identical to the maximum fill level margins presented in Figure 11. The reduction in rigid-body inertia due to fuel loss will not have a significant affect on controller stability.

Simulation of the Triana Thruster Controller

Figure 15: Nominal and True Step Response (72% FL)

The closed loop step response for the 72% in shown in Figure 15. The true dynamics show a higher overshoot and slower settling time than the idealized nominal loop. The differences in response are minor since the slosh mode is mostly cancelled by the complex zeros, although the slosh dynamics double the settling time. Both traces show the effects of the delay's right-half plane zeros as an initial excursion in the wrong direction.

Conclusions

This paper examined the effects of slosh dynamics on the Triana thruster-based attitude control. A first order model of the fuel slosh dynamics was developed. Slosh parameters were derived from empirical data. Transfer functions describing the slosh dynamics were developed. Continuous linear analysis was used to calculate the nominal and true stability margins, where nominal considered only rigid body dynamics whereas the true plant included the slosh dynamics. In both cases, the gain and phase margins were ample to meet requirements. This result is mainly due to the tank diaphragm. Examination of the true plant shows that the slosh mode was successfully suppressed: The second order slosh mode is approximately cancelled by a set of complex zeros produced by the diaphragm damping terms, proving that the presence of the diaphragm mitigates the effects of fuel slosh. More importantly, a comparison of unsuppressed fuel slosh model developed by Bryson and the damped slosh model developed here showed the attitude loop would be unstable without the slosh suppression provided by the bladder.

The true and nominal plants were studied at the beginning of life mass properties and tank fill level. The actual open loop transfer function will vary over the mission life as the tank empties, shifting mass properties and tank fill level. Further analysis quantified robustness by applying the multiplicative uncertainty model to create a boundary that must be satisfied by the closed loop frequency response. By varying fill level, the evolution of the robustness boundary was quantified. In all cases, the fuzzy ball stability criteria guaranteed stability of the true closed loop.

It is concluded that fuel slosh will not adversely effect the controller over the life of the mission because of the slosh suppression provided by the tank diaphragm. Indeed, analysis shows that without this propellant management device, the control system would be inherently unstable, proving the tank diaphragm is a crucial part of system stability.

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